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THE EFFECT OF GRAVITY INCLINATION ON HYDROMAGNETIC NON-LINEAR CHEMOTACTIC BIOCONVECTION

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ABSTRACT

This paper deals with the study of the effects of external magnetic field and gravity inclination on the chemotactic bacterial bioconvection by considering a Continuum model. Chemotaxis causes cells to swim out of the plume because the high concentration of the cells constituting the plumes leads to a lower concentration of oxygen in the surrounding fluid. There are cases where the layer is no longer vertical. In such cases, the gravity inclination plays a significant part. A similarity solution is found for the plume in which the cell flux and the volume flux could be matched to those in the boundary layer and also outside the suspension regions. Axisymmetric plumes are formed by applying two scales one with respect to the radial co-ordinate and the other with respect to the similarity variable. The effects of magnetic field and gravity inclination are remarkable, encouraging and the computed results are in excellent agreement with those of hydrodynamic case in the limiting case.

KEYWORDS: Axisymmetric Plumes, Bacterial Bioconvection, Cell Concentration, Chemotactic, Gravity Inclination, Similarity Solution

1. INTRODUCTION

World's major portion consists of biomass and bioconvection is the spontaneous formation of patterns in suspensions of swimming microorganisms due to their tactic nature viz. oxytactic, gyrotactic, chemotactic etc., The microorganisms exhibiting bioconvection have the following key features: They are denser than water and they swim upwards due to their tactic nature. This leads to an unstable situation in the system and thus an overturning instability develops leading to pattern formation [1][2][3]. The study of hydromagnetic convection is of great practical interest since Magnetic field has a strong influence on the system in many real time situations. .

Some literatures pertaining to bioconvection in deep chambers are [4][5]. The study of such a phenomenon has a variety of applications in biological and physiological problems. Further, chemotaxis and oxygen consumption are important in setting up the basic state and soon after, the resulting plumes are entirely buoyancy driven and the cells are merely advected. In such cases, the velocity would vary across the plume [6][7]. Experiments on bioconvection containing suspensions of bacteria (Bacillus Subtilis) have revealed the formation of Falling plumes when the system becomes unstable [8]. The present work investigates the nonlinear Hydromagnetic bioconvection to study the effect of magnetic field on the formation of falling plumes (Axisymmetric) where the oxygen consumption and chemotaxis are important in a gravity inclined environment. The model constituted the quasi – steady situation in which an upper boundary layer containing a high concentration of bacteria feeds a falling plume of cell-rich fluid. The suspension is divided into three separate regions, a cell-rich upper boundary layer of known thickness λ, a falling plume of unknown width ε which also contained a high concentration of bacteria and the fluid outside the plume which had to circulate in order to conserve mass.

Here, the assumption of the axisymmetric nature of the plume reduced the 3D-problem to 2D-problem [9]. Not much literature is available in this direction. The solutions were obtained by a Fast Computational Technique.

2. MATHEMATICAL FORMULATION

The bacterial suspension (Bacillus Subtilis) contained in a deep chamber reveal the development of a thin upper boundary layer of cell-rich saturated fluids which becomes unstable, leading to the formation of falling plumes which is a complex phenomenon. This was used as a basis for our mathematical model. The whole suspension is under the influence of uniform magnetic field.

The dimensionless governing equations are:

The equation of cell conservation

$$\frac{\partial N}{\partial t} = \nabla.\left[H(\theta)\nabla N - UN - H(\theta)\gamma N \nabla\theta\right] \tag{1}$$

The Navier – Stokes equation (with Boussinesq approximation)

$$Sc^{-1}\left[\frac{\partial U}{\partial t} + (U.\nabla)U\right] = -\nabla P_e + \nabla^2 U + \Gamma N \vec{K} - B^*(H.\nabla)H \tag{2}$$

Equation (2) in the component form is

$$Sc^{-1}\left[\frac{\partial U}{\partial t} + \left(U\frac{\partial}{\partial R} + W\frac{\partial}{\partial Z}\right)U\right] = -\frac{\partial}{\partial R}P_e + \left[\frac{1}{R}\frac{\partial U}{\partial R} + \frac{\partial^2 U}{\partial R^2} + \frac{\partial^2 U}{\partial Z^2}\right] + \Gamma N \sin\delta^* - B^*\left[H_x\frac{\partial}{\partial R} + H_z\frac{\partial}{\partial Z}\right]H_x \tag{2a}$$

$$Sc^{-1}\left[\left(U\frac{\partial}{\partial R} + W\frac{\partial}{\partial Z}\right)W\right] = -\frac{\partial}{\partial Z}P_e + \left[\frac{1}{R}\frac{\partial W}{\partial R} + \frac{\partial^2 W}{\partial R^2} + \frac{\partial^2 W}{\partial Z^2}\right] + \Gamma N \cos\delta^* - B^*\left[H_x\frac{\partial}{\partial R} + H_z\frac{\partial}{\partial Z}\right]H_z \tag{2b}$$

The equation of oxygen concentration

$$\frac{\partial\theta}{\partial t} = \nabla.\left[\delta\nabla\theta - U\theta - H(\theta)\delta\beta N\right] \tag{3}$$

The conservation of mass

$$\nabla.U = 0 \tag{4}$$

The magnetic induction equation

$$\frac{\partial H}{\partial t} + (U.\nabla)H = (H.\nabla)U + P_m\nabla^2 H \tag{5}$$

The variables are non – dimensionalized by using the following scales:

$$\nabla = \frac{1}{h}\nabla,\ N = \frac{\tilde{N}}{N_0},\ \theta = \frac{\tilde{C} - C_{min}}{C_0 - C_{min}},\ D_N = D_{N0}H(\theta),\ V = bV_s H(\theta)\nabla\theta,\ K = K_0 H(\theta) bV_s,$$

$$t = \frac{TD_{N0}}{h},\ U = \frac{\tilde{U}h}{D_{N0}},\ H = \frac{\tilde{H}}{H_0}$$

Where h : width of the chamber, V_s : it has dimensions of velocity, N_0 : initial cell concentration, U : saturated fluid velocity, $\vec{K}$: the inclination vector $= (\sin\delta^*, 0, \cos\delta^*)$, δ^* : gravity inclination to the vertical D_{N0}, K_0 : constants, D_N : the cell diffusivity, θ : the oxygen concentration, C_0 : the initial concentration, $H(\theta)$: the step function, T : the time, H_0 : the constant magnetic field.

2.1 Dimensionless Parameters

$$\beta = \frac{K_0 N_0 h^2}{D_c(C_0 - C_{min})}, \gamma = \frac{bV_s}{D_{N0}}, \delta = \frac{D_c}{D_{N0}}, B^* = \frac{\mu^* H_0^2 h^2}{\rho_w \nu D_{N0}}, \Gamma = \frac{\upsilon N_0 g h^3(\rho_c - \rho_w)}{\nu D_{N0}\rho_w}, Sc = \frac{\nu}{D_{N0}}, P_m = \frac{\nu_m}{D_{N0}}$$

Where β: strength of oxygen consumption relative to its diffusion, γ: measures the relative strength of directional and random swimming, δ: ratio of oxygen diffusivity to cell diffusivity, Γ: Bio – Rayleigh number, Sc: Schmidt number, P_m: magnetic prandtl number, ν: kinematic viscosity of the fluid, ρ_c, ρ_w: densities of cell and water, g: acceleration due to gravity, υ: volume of the cell, B^*: modified Hartmann number, μ^*: magnetic permeability, ν_m: magnetic viscosity of the fluid.

2.2 Boundary Conditions

- No slip condition at Z = 1 (bottom of the chamber)
- Stress free condition at the upper surface of the chamber i.e., at Z= 0
- The vertical components of velocity vanish at both the boundaries
- Zero cell – flux at both the boundaries
- Zero oxygen flux at the bottom surface and C = C_0 at the free surface
- H = 0 at both the boundaries
- The vertical components of velocity vanish at both the boundaries Mathematically,

$$\text{At } Z = 0,\ U.\hat{Z} = 0,\quad \frac{\partial^2}{\partial Z^2}(U.\hat{Z}) = 0,\quad \theta = 1,\quad H(\theta)\frac{\partial N}{\partial Z} - \gamma N H(\theta)\frac{\partial \theta}{\partial Z} = 0,\quad (H.\hat{Z}) = 0 \tag{6a}$$

$$\text{At } Z = 1,\quad U.\hat{Z} = 0,\quad U \times \hat{Z} = 0,\quad \frac{\partial \theta}{\partial Z} = 0,\quad \frac{\partial N}{\partial Z} = 0,\quad H.\hat{Z} = 0 \tag{6b}$$

3. AXISYMMETRIC PLUMES USING RADIAL CO-ORDINATES

In the plume, the radial co-ordinate is scaled as,

$$R = \varepsilon r,\quad 0 < \varepsilon < 1 \tag{7}$$

Rescaling: $N = N_A n,\ \theta = 1 + C_A C,\ W = W_A w,\ U = \varepsilon W_A u,\ P = P_A p$ (8)

Where N_A, C_A, W_A, and P_A are scale factors.

Using (7) and (8) the axisymmetric governing equations (neglecting $O(\varepsilon^2)$ terms) are as follows,

$$\frac{1}{r}\frac{\partial n}{\partial R} + \frac{\partial^2 n}{\partial r^2} = \varepsilon^2 W_A\left[u\frac{\partial n}{\partial r} + w\frac{\partial n}{\partial z}\right] + \gamma C_A\left[\frac{\partial n}{\partial r}\frac{\partial C}{\partial r} + \frac{n}{r}\frac{\partial C}{\partial r} + n\frac{\partial^2 C}{\partial r^2}\right] \tag{9}$$

$$\varepsilon^2 W_A Sc^{-1}\left[u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial Z}\right] = \frac{-P_A}{W_A}\frac{\partial p}{\partial r} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \varepsilon^2 B^*\frac{H_A^2}{W_A}\left[h_x\frac{\partial h_x}{\partial r} + h_z\frac{\partial h_x}{\partial Z}\right] + \varepsilon\gamma\frac{N_A}{W_A}n\sin\delta^* \tag{10}$$

$$\varepsilon^2 W_A Sc^{-1}\left[u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial Z}\right] = -\varepsilon^2\frac{P_A}{W_A}\frac{\partial p}{\partial Z} + \varepsilon^2\frac{N_A}{W_A}\Gamma n\cos\delta^* + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - \varepsilon^2 B^*\frac{H_A^2}{W_A}\left[h_x\frac{\partial h_z}{\partial r} + h_z\frac{\partial h_z}{\partial Z}\right] \tag{11}$$

$$\frac{\varepsilon^2 W_A}{\delta}\left[u\frac{\partial C}{\partial r} + w\frac{\partial C}{\partial z}\right] + \frac{\varepsilon^2\beta N_A n}{C_A} = \frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} \tag{12}$$

$$\frac{u}{r} + \frac{\partial u}{\partial r} + \frac{\partial w}{\partial Z} = 0 \tag{13}$$

$$\varepsilon^2 W_A\left[u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial Z}\right] = \varepsilon^2 H_A\left[h_x\frac{\partial u}{\partial r} + h_z\frac{\partial u}{\partial Z}\right] + P_m\frac{H_A}{W_A}\left[\frac{1}{r}\frac{\partial h_x}{\partial r} + \frac{\partial^2 h_x}{\partial r^2}\right] \tag{14}$$

$$\varepsilon^2 W_A\left[u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial Z}\right] = \varepsilon^2 H_A\left[h_x\frac{\partial w}{\partial r} + h_z\frac{\partial w}{\partial Z}\right] + P_m\frac{H_A}{W_A}\left[\frac{1}{r}\frac{\partial h_z}{\partial r} + \frac{\partial^2 h_z}{\partial r^2}\right] \tag{15}$$

Here, $W_A = \varepsilon^{-2}$ (to retain the advection term), $C_A = \frac{2}{\gamma}$ (to retain the chemotaxis term), $N_A = \frac{\lambda}{\varepsilon^2}$ (to retain the oxygen consumption term in 12), $\Gamma = O(\lambda^{-1}\varepsilon^{-2})$ (to retain the buoyancy term in 11), $P_A = \varepsilon^{-4}$ (to retain the pressure term in 11), $H_A = \varepsilon^{-2}$ (to retain the induction term in 15).

This leads to $\frac{\partial p}{\partial r} = 0$, hence p = p (Z) in (10). (16)

Substituting for C_A, W_A and N_A in (9) and (12) we get the following equations,

$$u\frac{\partial n}{\partial r} + w\frac{\partial n}{\partial Z} + 2\frac{\partial n}{\partial r}\frac{\partial C}{\partial r} + 2\frac{n}{r}\frac{\partial C}{\partial r} + 2n\frac{\partial^2 C}{\partial r^2} - \frac{1}{r}\frac{\partial n}{\partial r} - \frac{\partial^2 n}{\partial r^2} = 0 \tag{17}$$

$$\frac{1}{\delta}\left[u\frac{\partial C}{\partial r} + w\frac{\partial C}{\partial Z}\right] + n\left[\frac{1}{r}\frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial r^2}\right] = 0 \tag{18}$$

Differentiating (11) w.r.t. r and substituting for N_A, W_A, H_A and Γ we get,

$$Sc^{-1}\left[\frac{\partial u}{\partial r}\frac{\partial w}{\partial r}+u\frac{\partial^2 w}{\partial r^2}+\frac{\partial w}{\partial r}\frac{\partial w}{\partial Z}+w\frac{\partial^2 w}{\partial r\partial Z}\right]=\tilde{\Gamma}\cos\delta^*\frac{\partial n}{\partial r}+\frac{\partial^3 w}{\partial r^3}+\frac{1}{r}\frac{\partial^2 w}{\partial r^2}-\frac{1}{r^2}\frac{\partial w}{\partial r}-$$
$$B^*\left[\frac{\partial h_x}{\partial r}\frac{\partial h_z}{\partial r}+h_z\frac{\partial^2 h_z}{\partial r^2}+\frac{\partial h_z}{\partial r}\frac{\partial h_z}{\partial Z}+h_z\frac{\partial^2 h_z}{\partial r\partial Z}\right] \tag{19}$$

Differentiating (15) w.r.t r and substituting for H_A, W_A we get,

$$\left[\frac{\partial u}{\partial r}\frac{\partial w}{\partial r}+u\frac{\partial^2 w}{\partial r^2}+\frac{\partial w}{\partial r}\frac{\partial w}{\partial Z}+w\frac{\partial^2 w}{\partial r\partial Z}\right]=\left[\frac{\partial h_x}{\partial r}\frac{\partial w}{\partial r}+h_x\frac{\partial^2 w}{\partial r^2}+\frac{\partial h_z}{\partial r}\frac{\partial w}{\partial Z}+h_z\frac{\partial^2 w}{\partial r\partial Z}\right]+$$
$$P_m\left[\frac{-1}{r^2}\frac{\partial h_z}{\partial r}+\frac{1}{r}\frac{\partial^2 h_z}{\partial r^2}+\frac{\partial^3 h_z}{\partial r^3}\right] \tag{20}$$

Now, imposing the boundary conditions:

$$\frac{\partial n}{\partial r}=0,\quad \frac{\partial C}{\partial r}=0,\quad u=0,\quad \frac{\partial w}{\partial r}=0 \tag{21a}$$

$$\text{Also, } r\to\infty,\quad n\to 0,\quad \frac{\partial C}{\partial r}\to 0,\quad w\to 0 \tag{21b}$$

3.1 Similarity Solution for Axisymmetric Case

In order to obtain a similarity solution [10][11] for (17), (18), (19), (20) the solution is posed in the form

$$h :: Z^a,\ w :: Z^b,\ n :: Z^c,\ C :: Z^d,\ u :: Z^{a+b+1} \tag{22}$$

(h: width of the plume, a = ½, b = 0, c = -1, d = 0)

Since h: $Z^{1/2}$, the similarity variable is defined as $\eta=\dfrac{r}{Z^{1/2}}$ (23)

Assuming the solution in the form

$$n=Z^{-1}H(\eta),\quad C=G(\eta),\quad \psi=ZF(\eta),\quad u=Z^{-1/2}\left(\frac{F}{\eta}-\frac{F'}{2}\right),\quad w=\frac{-F'}{\eta} \tag{24}$$

(ψ : Stream functions, Primes denote differentiation w.r.t η)

Substituting these into (17) and integrating once w.r.t η with the boundary conditions at η = 0, we get the following equation:

$$HF+2\eta HG'-\eta H'=0. \tag{25}$$

Substituting into (18):

$$\eta G''+G'-\frac{1}{\delta}G'F-\eta H=0. \tag{26}$$

Substituting into (19):

$$\frac{1}{\eta}F''' - \frac{1}{\eta^2}F'' + \frac{1}{\eta^3}F' + Sc^{-1}\left[\frac{1}{\eta^3}FF' - \frac{1}{\eta^2}FF''\right] - \tilde{\Gamma}\cos\delta^* H - B^*\left[\frac{1}{\eta^3}MM' - \frac{1}{\eta^2}MM'\right] = 0. \tag{27}$$

Substituting into (20):

$$M = F \tag{28}$$

The boundary conditions are ,

$$\text{At } \eta = 0 : H' = G' = \frac{F}{\eta} - \frac{F'}{2} = \frac{F'}{\eta^2} - \frac{F''}{\eta} = 0$$

$$\text{At } \eta \to \infty : H \to 0,\ G' \to 0,\ \frac{F'}{\eta} \to 0,\ \frac{M'}{\eta} \to 0 \tag{29}$$

4. SOLUTION

For $\gamma \neq 0$, CFD technique is employed. However for $\gamma = 0$ (i.e., when the chemotaxis is unimportant in the plume) analytical solutions are possible with $\beta = O(1)$, $C_o = 1$, $N_o = \varepsilon^{-2}$ and $\Gamma = \varepsilon^{-2}\tilde{\Gamma}$. Following [4] [9] the solutions for the equations (23, 24, 25, 26) are found to be (see table 1)

$$B = \frac{-12}{1 + Sc^{-1} - B^*} \quad \text{and} \quad C = \left[\frac{192A^2}{1 + Sc^{-1} - B^*}\right]\frac{1}{\tilde{\Gamma}\cos\delta^*}$$

Table 1: Solutions for F, H, G'

At $Sc = 1,\ \delta = 1$	At $Sc = 2,\ \delta = 1$	At $Sc = 1,\ \delta = 1$ when $\delta^* = 0, B^* = 0$
$F = \frac{-12A\eta^2}{(2 - B^*)(1 + A\eta^2)}$	$F = \frac{-24A\eta^2}{(3 - 2B^*)(1 + A\eta^2)}$	$F = \frac{-6A\eta^2}{1 + A\eta^2}$
$H = \frac{192A^2}{\tilde{\Gamma}\cos\delta^*(2 - B^*)(1 + A\eta^2)^{\frac{6}{2-B^*}}}$	$H = \frac{384A^2}{\tilde{\Gamma}\cos\delta^*(3 - 2B^*)(1 + A\eta^2)^{\frac{12}{3-2B^*}}}$	$H = \frac{96A^2}{\tilde{\Gamma}(1 + A\eta^2)^3}$
$G' = \frac{96A^2\eta}{\tilde{\Gamma}\cos\delta^*(2 - B^*)(1 + A\eta^2)^{\frac{6}{2-B^*}}}$	$G' = \frac{192A^2\eta}{\tilde{\Gamma}\cos\delta^*(3 - 2B^*)(1 + A\eta^2)^{\frac{12}{3-2B^*}}}$	$G' = \frac{48A^2\eta}{\tilde{\Gamma}(1 + A\eta^2)^3}$

Also solutions satisfy the boundary conditions at $\eta = 0$ and $\eta \to \infty$.

$$A^2 = \frac{Q\tilde{\Gamma}\cos\delta^*(2 - B^*)(8 - B^*)}{2304} \text{ for } Sc = 1 \text{ and } A^2 = \frac{Q\tilde{\Gamma}\cos\delta^*(3 - 2B^*)(15 - 2B^*)}{9216} \text{ for } Sc = 2$$

Since $\int_0^\infty [HF'd\eta = -Q]$

In table 1, the first two columns correspond to the hydromagnetic case while the third column corresponds to the hydrodynamic case in the absence of gravity inclination [8] . Our results are in excellent agreement with those of the hydrodynamic case in the absence of gravity inclination.

5. RESULTS AND DISCUSSIONS

In this study, the deep chamber experiment [8] has been modeled in three separate regions:

a) an upper boundary layer of depth λ_R b) a falling plume of width ε c) The region outside the plume.

In sections 3 and 4, solutions for the cell and the oxygen concentration and the fluid velocity in the upper boundary layer are determined under the influence of a uniform vertical magnetic field and gravity inclination. The solutions are found to depend on the parameters like, Sc (Schmidt number), Q (the cell flux), $\tilde{\Gamma}$ (Bio-Rayleigh number), B^* (Magnetic Parameter), δ^* (gravity inclination) and δ (diffusivity ratio). The computations are performed using the MATLAB tool; the computed results are presented through graphs in Figures 1 to 13.

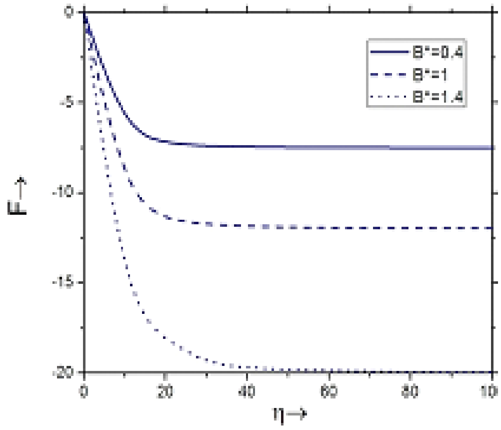

Figure1: $F\ vs\ \eta$

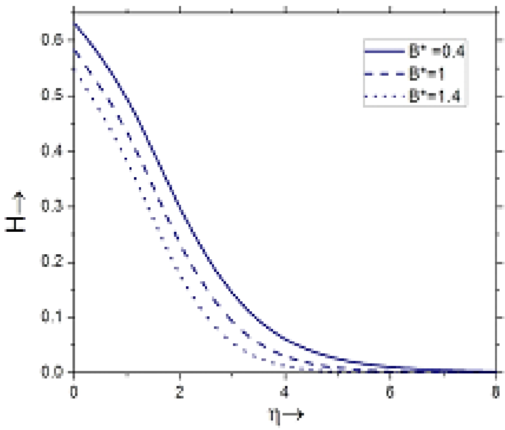

Figure 2: $H\ vs\ \eta$

The following observations are made: The effect of variation in the magnetic parameter B^* on the profiles of velocity $w = \left(-F'/\eta\right)$, the cell concentration H and the oxygen concentration G is studied for the values, $Sc = \delta = \tilde{\Gamma} = Q = 1.0$. Here, the oxygen concentration is considered as $\theta = \left[1 + \left(2/\gamma\right)G(\eta)\right]$ where γ is assumed to be always positive so that, all the bacteria are active. There is no effect of gravity inclination on F. In figures 2 and 3 the effect of gravity inclination with variation in magnetic parameter B^* is studied for $Sc = \delta = \tilde{\Gamma} = Q = 1.0$ and $\delta^* = 50^0$.

In Figure.1, the effect of similarity variable η on the F profile are shown. F decreases enormously and remains constant as $\eta \to \infty$ for all values of B^*. Further, F is negative and its value is highest in the hydrodynamic case. In other words, F increases in absolute value as B^* increases, and the vertical fluid velocity w, at the center of the plume increases indicating that the horizontal fluid flow into the plume increases.

In Figure.2 the effect of similarity variable on H profile is shown, it reveals that, as B^* decreases the cell concentration in the plume increases as expected. Physically it means that, the higher the concentration of the cells, the greater is the consumption of oxygen which means that the oxygen concentration at the center of the plume decreases. In Figure.3 the effect of similarity variable on G' profile is shown. Clearly the width of the plume decreases as B^* increases. The oxygen concentration is more in the hydrodynamic case (B^* = 0) when compared to the hydro magnetic case ($B^* \neq 0$).

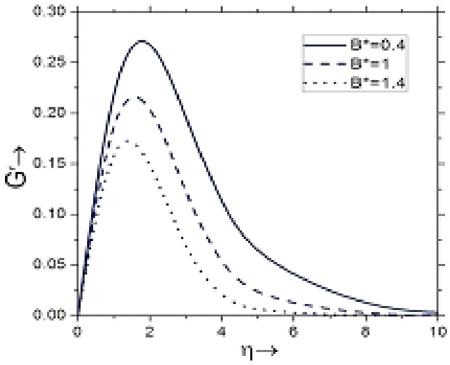

Figure 3: G' $vs\,\eta$

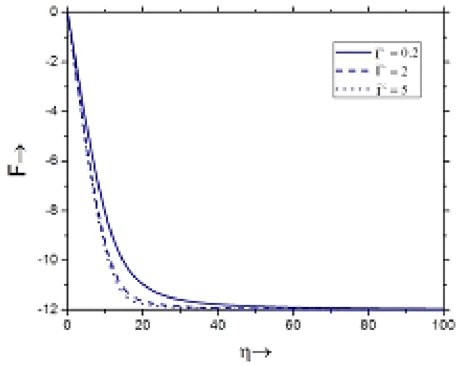

Figure 4: F $vs\,\eta$

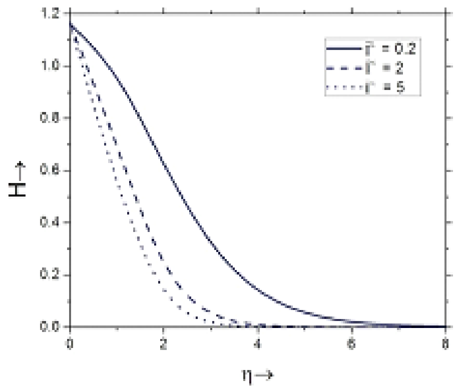

Figure 5: H $vs\,\eta$

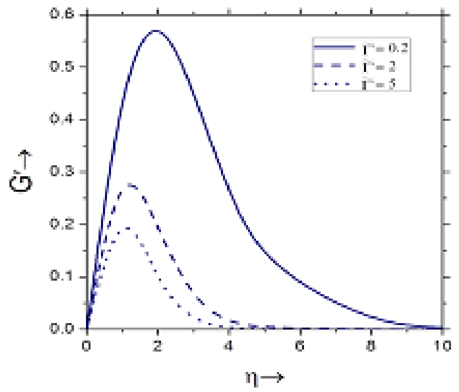

Figure 6: G' $vs\,\eta$

Figures 4, 5, 6. represent the effect of variation of $\tilde{\Gamma}$ on the profiles F, H, G' for fixed values of the parameters $Sc = 1, Q = 2, B^* = 1, \delta^* = 50^0$. The values of $\tilde{\Gamma}$ considered are 0.2, 2 and 5. It is observed that there is an enhancement in the values of F, H, G' when compared to the uninclined case [8]. The effect of buoyancy becomes important when $\tilde{\Gamma}$ is large. The cell concentration is more for small values of $\tilde{\Gamma}$ and the plume becomes narrower for large $\tilde{\Gamma}$. When the cell concentration in the centre of the plume increases, the plume becomes narrower, accordingly the oxygen profile becomes narrower and the oxygen concentration at the center of the plume increases. The consumption of oxygen will be more. Therefore, the velocity of the fluid in the Center of the plume will be larger when the buoyancy force is dominant. But w→0 more rapidly than for small values of $\tilde{\Gamma}$. The decrease in M for the increase in $\tilde{\Gamma}$ indicates that less fluid is entrained by the plume.

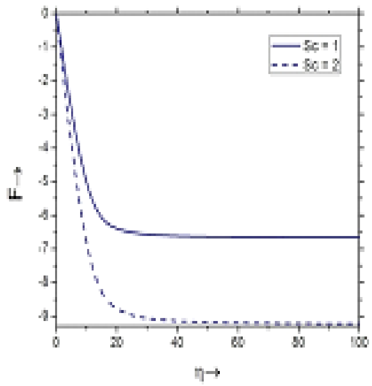

Figure 7: F $vs\,\eta$

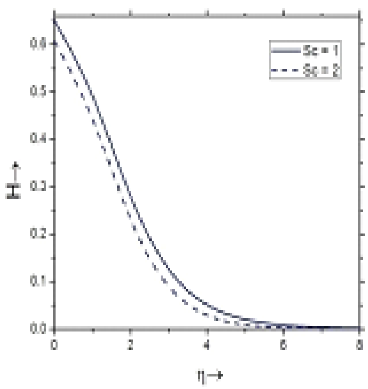

Figure 8: H $vs\,\eta$

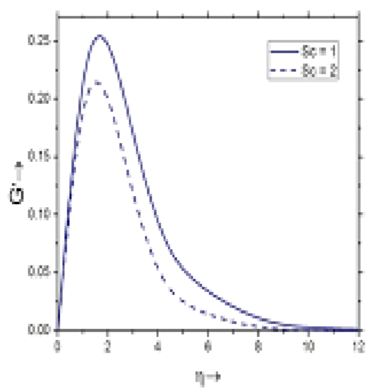

Figure 9: G' $vs\,\eta$

Figures 7, 8, 9. present the graph of the profiles F, H, G' when the values of Sc (= 1, 2) are varied. The other parameters have fixed values viz., $\delta = 1, Q = 1, B^* = 0.2, \delta^* = 10^0$. It is observed that as in the hydrodynamic case $\left(B^* = 0\right)$, the variation in Sc has a significant effect on the behavior of the profiles. There is a drastic change in the values of F for Sc = 1 and 2. As Sc increased, F decreases rapidly and F → a constant value as $\eta \to \infty$ as expected. The cell concentration will be more for Sc = 2 and accordingly the oxygen consumption in the plume will be more and there will be a reduction in the oxygen concentration in the plume for any value of δ^*.

Finally figures 10, 11 present the effect of δ^* on the profiles of H and G'. Both H and G' increase with δ^* & the general behavior is the same (as in δ^*= 0). The same behavior is exhibited even in the hydrodynamic case (Figures 12, 13).

Finally it is concluded that the governing dimensionless parameters viz., $\delta^*, B^*, Sc, \tilde{\Gamma}, \delta$ and Q have a strong influence on the bioconvective system considered. It is observed that (i) the governing dimensionless parameters have a remarkable effect in the hydrodynamic as well as in the hydromagnetic cases (ii) the qualitative nature of the profiles is almost the same in both the cases but there is a drastic difference in the quantitative nature of the profiles.

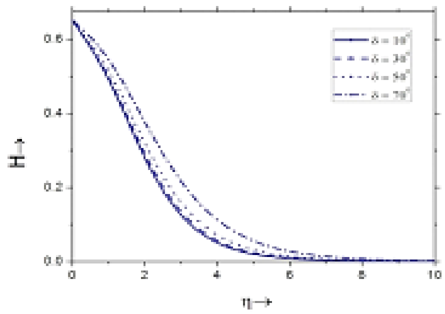

Figure10: H $vs\,\eta$

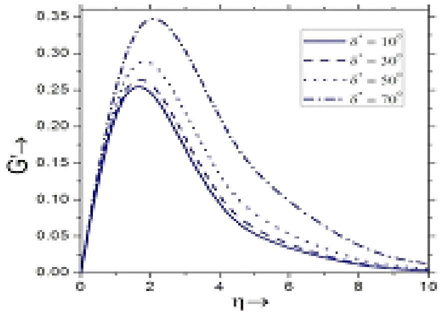

Figure11: G′ $vs\,\eta$

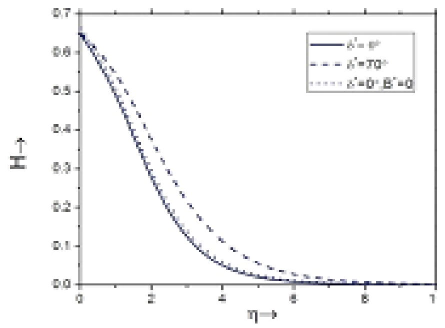

Figure12: H $vs\,\eta$

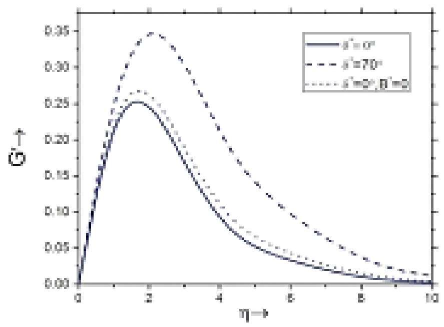

Figure13: G′ $vs\,\eta$

Figures clearly indicate the strong influence of the magnetic parameter on the present bioconvective system, these clearly suggest that the plume convection could be suppressed or enhanced through the proper choice of the magnetic parameter. The results are in excellent agreement with the hydrodynamic case in the presence and absence of inclination. From the present study it is possible to retrieve the results pertaining to nonlinear chemotactic bioconvection for the following cases viz., non-linear hydrodynamic chemotactic bioconvection in the presence and absence of gravity inclination and non-linear hydromagnetic chemotactic bioconvection in the presence and absence of gravity inclination. Therefore, it is possible to suppress or enhance chemotactic bioconvection by a suitable choice of the governing dimensionless parameters.

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